

Full Length Research Paper

Application of the Analytic Hierarchy Process (AHP) for locating fire stations: Case Study Maku City

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Abstract

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Decision making problem is the process of finding the best option from all of the feasible alternatives. In almost all such problems the multiplicity of criteria for judging the alternatives is pervasive. That is, for many such problems, the decision maker wants to solve a multiple criteria decision making (MCDM) problem. A survey of the MCDM methods has been presented by Hwang and Yoon. The analytic hierarchy process (AHP) is one of the extensively used multi-criteria decision-making methods. One of the main advantages of this method is the relative ease with which it handles multiple criteria. In addition to this, AHP is easier to understand and it can effectively handle both qualitative and quantitative data. The use of AHP does not involve cumbersome mathematics. AHP involves the principles of decomposition, pair wise comparisons, and priority vector generation and synthesis. Though the purpose of AHP is to capture the expert's knowledge. In the present research, combination of indexing system Method with Analytical Hierarchy Process has been applied to assess locating Fire stations. By this process, classification and qualification of the numerous types of regions would be accessible. The findings of the research show that Region 2 with (0/352) point promotes in first rank among 5 studied Regions and thus it is the most appropriate Region, in contrast Region 5 with (0/112) point goes down to the last rank. Regions (1, 3, 4) with (0/191, 0/171, 0/170) points are located in next ranks.

Keywords: Analytic Hierarchy Process, Location, Fire stations, Maku.

INTRODUCTION

Today, excess density of population in city and it is increasing growth in bulk is led to demand and attention to urban development. Demand for urban development is one of the most important issues against human in future. Therefore, to solve this problem and obstacles, safety system of city should be developed along this to cover whole city. The most important problem about the services of fire stations is the inappropriate distribution of stations and restricted function area of present stations. So, qualities and quantities distribution of stations is investigated scientifically and professionally. Using

traditional methods planning fire stations for services mean wasting papers and time. The analytic hierarchy process (AHP) is a structured technique for dealing with complex decisions that was developed by Thomas L. Saaty in the 1980 year. It provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions. The base of this model is comparing variables by pair wise by Matrix relationship. In this way, pair wise of the effective variables on the concrete

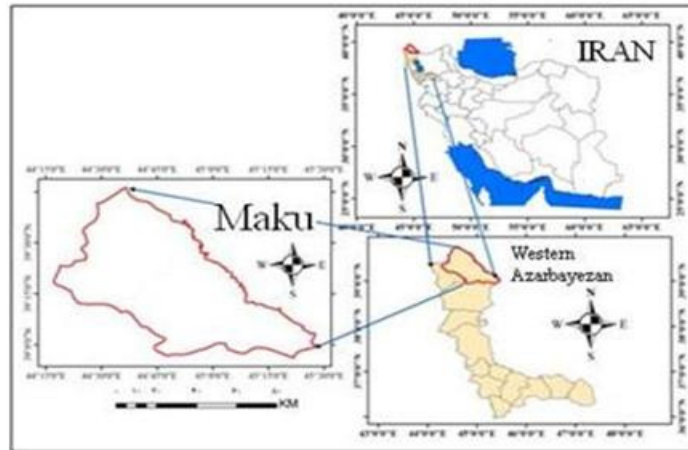


Figure 1. Mathematical situation of studied area.

Pavement were considered and based on relative weights the output was extent The Analytic Hierarchical Process is a structured technique for dealing with complex decisions. Rather than prescribing a correct decision, the AHP helps decision makers to find a solution that best suits their goal and their understanding of the problem. It is a process of organizing decisions that people are already dealing with, but trying to do in their heads. The AHP was developed by Thomas L. Saaty in the 1970s and has been extensively studied and refined since then. It provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions. Users of the AHP first decompose their decision problem into a hierarchy of more easily comprehended sub-problems, each of which can be analyzed independently. The elements of the hierarchy can relate to any aspect of the decision problem tangible or intangible, carefully measured or roughly estimated, well or poorly-understood anything at all that applies to the decision at hand. Once the hierarchy is built, the decision makers systematically evaluate its various elements by comparing them to one another two at a time, with respect to their impact on an element above them in the hierarchy. In making the comparisons, the decision makers can use concrete data about the elements, or they can use their judgments about the elements' relative meaning and importance. It is the essence of the AHP that human judgments, and not just the underlying information, can be used in performing the evaluations. The AHP converts these evaluations to numerical values that can be processed and compared over the entire range of the problem. A numerical weight or priority is derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. This capability distinguishes the AHP from other decision making techniques. In the final step of the

process, numerical priorities are calculated for each of the decision alternatives. These numbers represent the alternatives' relative ability to achieve the decision goal. Thus, they allow a straightforward consideration of the various courses of action. There are many examples of applications of multiple criteria decision making in literature (For instance: The evaluation of service quality (Mousavi et al., 2010); Intercompany comparison (Limon and Martinez, 2006); The applications in aggregate production planning (Harker and Vargas, 1987), Facility location selection (Mianabadi and Afshar, 2008) and large scale nonlinear programming (Wang and Liang, 2004). The modifications proposed in this paper can be implemented in all real world applications of Fuzzy TOPSIS, Krishnamurthy et al. (1995, 1996) used RS and GIS techniques to find a suitable position for artificial recharge of ground water in India. Also, they investigated the effects of geomorphologic and geological factors on the behavior of ground water and stated that there is a special unevenness in each area for recharge of ground water. Saraf and Choudhury (1998) used remote sensing capabilities in extracting different layers like land usage, geomorphology, vegetation, and their integration in GIS environment to determine the most suitable area for artificial recharge of ground water. Mahdavi (1997, 16) investigated water management and artificial recharge of ground water in Jourm city and indicated that controlling usage and recharge of water tables by the watershed management is the main management technique. In this study, we tried to locate optimized fire station in Maku city using Analytic Hierarchy Process

METHODS AND MATERIALS

Mathematical situation of studied area

Maku city being situated in the north part of Western Azarbayezan province, Maku city is bounded by $39^{\circ}, 00'$

latitude to 18°, 00' north latitude and 44°, 31' longitude. Globally, Maku is located at 2560 meter height above sea level. Distance from city to center of province is 280km and distance from city to Tehran is 850km.

RESEARCH METHODOLOGY

Discovering the main components and indicators of environmental sustainability in a comprehensive and organized way for evaluation and assessing sustainability in urban areas is multi-criteria techniques in the structure of integrated satiability assessment. Such technique helps the users to understand the results of integrated assessment like evaluating policy aims and applying these results in a system and proposed decision making for sustainable development. There are various tools in the field of multi-criteria decision making models which could help planners and policy makers to solve decision difficulties with respect to different and contradict opinions. These models are TOPSIS, SAW, LINMAP, AHP, ANP, ELECTRE, Linear Assignment, PROMETHEE I and II, Compromise Programming and other methods. In the present paper AHP was applied which is a concordance subset. Coordinate subset is the third subset of compensatory models in MADM which their output would be a set of ranks so that provide necessary coordination in a most proper way. This subset includes AHP and linear assignment methods. The data and information of the research were collected by reviewing different documental proofs in the related offices. Also a field survey was conducted to gather main research data and information by completing questionnaires. Then this data analysis with AHP technique.

Theoretical basis

Analytic hierarchy process (AHP), as a very popular multiple criteria decision making (MCDM) tool, has been considerably criticized for its possible rank reversal phenomenon, which means changes of the relative rankings of the other alternatives after an alternative is added or deleted. If the weights or the number of criteria are also changed, then rankings might be reversed. Such a phenomenon was first noticed and pointed out by (Belton and Gear, 1983), which leads to a long-lasting debate about the validity of AHP (Dyer, 1990; Harker and Vargas, 1987; Wang and Liang, 2004; Saaty et al., 1983; Stewart, 1992; Troutt, 1988; Vargas, 1994; Watson and Freeling, 1982; Watson and Freeling, 1983), especially about the legitimacy of rank reversal (Forman, 1990; Millet and Saaty, 2000; Saaty, 1987; Saaty, 1987; Saaty and Vargas, 1984; Schoner et al., 1992). In order to avoid the rank reversal, (Belton and Gear, 1983) suggested

normalizing the eigenvector weights of alternatives using their maximum rather than their sum, which was usually called B–G modified AHP. (Saaty and Vargas, 1984) provided a counterexample to show that B– G modified AHP was also subject to rank reversal. (Belton and Gear, 1985) argued that their procedure was misunderstood and insisted that their approach would not result in any rank reversal if criteria weights were changed accordingly. (Schoner and Wedley, 1989) presented a referenced AHP to avoid rank reversal phenomenon, which requires the modification of criteria weights when an alternative is added or deleted (Schoner et al., 1993) also suggested a method of normalization to the minimum and a linking pin AHP (see also (Schoner et al., 1997)), in which one of the alternatives under each criterion is chosen as the link for criteria comparisons and the values in the linking cells are assigned a value of one, with proportional values in the other cells. Barzilai and Golany (Barzilai et al., 1987) showed that no normalization could prevent rank reversal and suggested a multiplicative aggregation rule, which replaces normalized weight vectors with weight–ratio matrices, to avoid rank reversal. Lootsma (Lootsma, 1993) and Barzilai and Lootsma (Barzilai and Lootsma, 1997) suggested a multiplicative AHP for rank preservation. Vargas (Mianabadi and Afshar, 2007) provided a practical counter-example to show the invalidity of the multiplicative AHP. Triantaphyllou (Triantaphyllou, 2001) offered two new cases to demonstrate that the rank reversals do not occur with the multiplicative AHP, but do occur with the AHP and some of its additive variants. Leung and Cao (Leung and Cao, 2001) showed that Sinarchy, a particular form of analytic network process (ANP), could prevent rank reversal. As an integrative view, the AHP now supports four modes, called Absolute, Distributive, Ideal and Supermatrix modes, respectively, for scaling weights to rank alternatives (Millet and Saaty, 2000; Saaty, 1986; Saaty, 1994; Saaty and Vargas, 1993). In the absolute mode, alternatives are rated one at a time and there is no rank reversal when new alternatives are added or removed. The distributive mode normalizes alternative weights under each criterion so that they sum to one, which does not preserve rank. The ideal mode preserves rank by dividing the weight of each alternative only by the weight of the best alternative under each criterion. The super matrix mode allows one to consider dependencies between different levels of a feedback network. More recently, Ramanathan (Ramanathan, 2006) suggested a DEAHP, which is claimed to have no rank reversal phenomenon. But in fact, it still suffers from rank reversal. Wang and Elhag suggested an approach in which the local priorities remained unchanged. So, the ranking among the alternatives would be preserved.

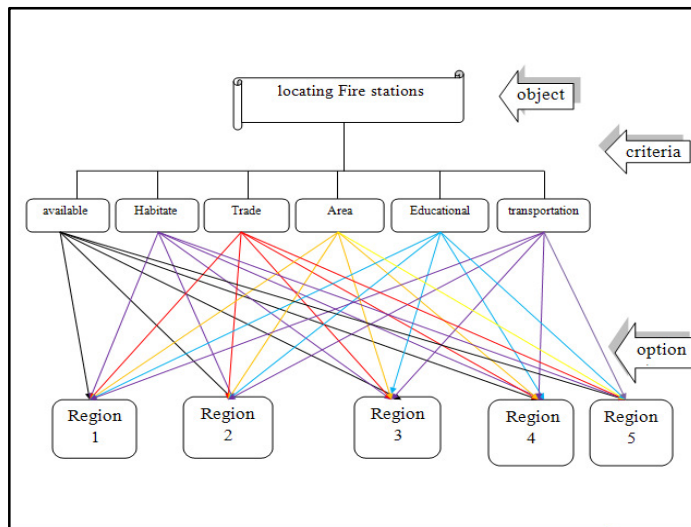


Figure 2. The process of hierarchical analytic

Table 1. Weighting the factors based on preference in paired comparison (Ghods Poor, 2009, 14)

Numerical values	Preferences (judging verbal)
9	Extremely preferred
7	Very strongly preferred
5	Strongly preferred
3	Moderately referred
1	Equally preferred
2·4·6·8	Intervals between strong preferences

Applying AHP technique for locating fire stations

Build the hierarchy

In the first action, the hierarchical structure of the investigated subject mariot we traced (Figure 1). In this figure we have a 3 level hierarchy includes the objective, criteria and options we face. Turning to a subject or issue the hierarchical structure is the most important part of the hierarchical analytic can be considered, because in this episode with the complex issues and difficult process of hierarchical analysis to make it plain that the mind and human nature to match. In other words, the process of hierarchical analytic of complex issues through its analysis to the minor elements that are linked together to form a hierarchy and communicate the main objective of the issue with the lowest hierarchical level is specified in the form of easier comes in.

The following criteria and explaining the importance of the factor criteria

To determine the coefficient of importance (weight) the following criteria and criteria for comparing two to two.

For example, for the purpose of the issue is that the criteria for access to locate are aware of the importance of more residential density or criteria? The basis of judging the comparative quantification of this table (Table 1) below that is based on it and according to the criteria for excellence to evaluate the severity of i relative to the criteria for a_{ij} , j. All the criteria are compared with each other mutually. In the process of analysis of the highest weight of the layer hierarchy is the effect that the highest awarded in the determination of the purpose. In other words, the information unit and weight criteria also had the highest based on the role it plays within the layers.

After the formation of paired comparison matrix, relative weights of criteria can be calculated. There are different methods to calculate the relative weight based on paired comparison matrix. The most important ones are the "least squares method, least squares logarithmic method, special vector method and approximate method. The special vector method is the most accurate one. In this method, W_i is determine in the equation 12:

$$A \times W = \lambda \max W \quad (20)$$

In this equation, λ and W are special amount and special vector of paired matrix respectively. If dimensions of matrix were larger, calculation would be too time-consuming. So, to calculate λ , the amount of Dtrmynal

Table 2. Quantity of I.I.R

n	1	2	3	4	5	6	7	...
I.I.R	0	0	0/58	0/9	1/12	1/24	1/32	...

λ A-matrix will be equaled to zero. Considering the greatest value of λ in equation (13), the amount of w_i is calculated. (2001, 315: Saaty).

$$A - \lambda_{\max} I = 0$$

Preparation of matarishai and narmalizah analyzed-invoices

Through the method of forming of weight to factors in the prevention of drug addiction and they set based on the importance of comparison-and narmalizah with matrix, rank 9 for parameters and options for 36-order form. In the next step, please refer any one of the values of matrix comparison-together and each element in the comparison-matrix-was divided up into its own column comparison-narmalizah matrices (1 relationship). Then the mean of the elements in each row of the matrix has been calculated in narmalizah the result will be the creation of weight vector (about 2).

$$r_{ij} = \frac{a_{ij}}{\sum_{i=1}^m a_{ij}}$$

$$W_i = \frac{\sum_{i=1}^n r_{ij}}{n}$$

The final rate-determining factors (priority and priority)

For this Act of the maratbi dynasty which led to the composition of the principle vector of priority taking into consideration all of the judgment of the maratbi dynasty at all levels, shall be used. In other words, the final score of each set of coefficients of the sum combination prevention options and determine the parameters to be fitted (3 relationship).

$$V_H = \sum_{k=1}^n W_k (g_{ij})$$

Calculation of adjustment and unadjustment

To calculate the rate adjustment, first must (A) compare the matrix-vector multiplication on the weight (W) to obtain an estimation of $\lambda_{\max} w$. In the other hand $A \times W = \lambda_{\max} W$. With the Division of the value of $\lambda_{\max} w$ in w calculated on the quantity of λ_{\max} . Then the amount of

"relationship indicator (4) will be calculated, (Ghodsi, 2008)

$$I.I. = \frac{\lambda_{\max} - n}{n-1}$$

"unjustment Rate is also calculated by this relation

$$I.R. = \frac{I.I.}{I.I.R.}$$

Quantity of I.I.R extracted from this table 2

DISCUSSION

The analytical hierarchy procedure (AHP) is proposed by Saaty (Saaty, 1980). AHP was originally applied to uncertain decision problems with multiple criteria, and has been widely used in solving problems of ranking, selection, evaluation, optimization, and prediction decisions. The AHP method is expressed by a unidirectional hierarchical relationship among decision levels. The top element of the hierarchy is the overall goal for the decision model. The hierarchy decomposes to a more specific criterion in which a level of manageable decision criteria is met (Mianabadi and Afshar, 2008). Under each criterion, sub-criteria elements related to the criterion can be constructed. The AHP separates complex decision problems into elements within a simplified hierarchical system (Limon and Martinez, 2006). The AHP usually consists of three stages of problem solving: decomposition, comparative judgment, and synthesis of priority. The decomposition stage aims at the construction of a hierarchical network to represent a decision problem, with the top level representing overall objectives and the lower levels representing criteria, sub criteria

and alternatives. With comparative judgments, expert users are requested to set up a comparison matrix at each hierarchy by comparing pairs of criteria or sub-criteria. Finally, in the synthesis of priority stage, each comparison matrix is then solved by an eigenvector (Wang and Liang, 2004) method for determining the criteria importance and alternative performance. The purpose of the AHP enquiry in this paper was to construct a hierarchical evaluation system based on the resource attributes and entity reputation. The results of AHP method for prioritize of concrete Pavement showed in

Table 3. Paired comparison table to the criteria according to the purpose

Related to locating Fire stations	Available	Habitata	Trade	Area	Educational	transportation	Wij
available	1	3	5	7	8	9	0.4482
Habitata	0.33	1	3	5	7	8	0.2542
Trade	0.2	0.33	1	3	5	7	0.1461
area	0.14	0.2	0.33	1	3	5	0.0810
educational	0.12	0.14	0.2	0.33	1	3	0.0447
transportation	0.11	0.12	0.14	0.2	0.33	1	0.0258
Sum	1.9	4.79	9.67	16.53	24.33	33	1

Inconsistency rate: 0/0322 (due to being less than 0/1 compatibility matrix indices are acceptable)

Table 4. Paired comparison table to the options according to the available

Related to available	Region 5	Region 4	Region 3	Region 1	Region 2	Wij
Region 5	1	3	5	7	9	0.4988
Region 4	0.33	1	3	5	7	0.2589
Region 3	0.33	0.2	1	3	5	0.1412
Region 1	0.14	0.2	0.33	1	3	0.0669
Region 2	0.11	0.14	0.2	0.33	1	0.0341
Sum	1.91	4.54	9.53	16.33	25	1

Inconsistency rate: 0/066 (due to being less than 0/1 compatibility matrix indices are acceptable)

Table 5. Paired comparison table to the options according to the trade

Related to trade	Region 5	Region 4	Region 3	Region 1	Region 2	Wij
Region 5	1	3	5	7	9	0.4988
Region 4	0.33	1	3	5	7	0.2589
Region 3	0.33	0.2	1	3	5	0.1412
Region 1	0.14	0.2	0.33	1	3	0.0669
Region 2	0.11	0.14	0.2	0.33	1	0.0341
Sum	1.91	4.54	9.53	16.33	25	1

Inconsistency rate: 0/066 (due to being less than 0/1 compatibility matrix indices are acceptable)

Table 6. Paired comparison table to the options according to the Habitata

Related to Habitata	Region 5	Region 4	Region 3	Region 1	Region 2	Wij
Region 5	1	3	5	7	9	0.4988
Region 4	0.33	1	3	5	7	0.2589
Region 3	0.33	0.2	1	3	5	0.1412
Region 1	0.14	0.2	0.33	1	3	0.0669
Region 2	0.11	0.14	0.2	0.33	1	0.0341
Sum	1.91	4.54	9.53	16.33	25	1

Inconsistency rate: 0/066 (due to being less than 0/1 compatibility matrix indices are acceptable)

Table 7. Paired comparison table to the options according to the Area

Related to Area	Region 5	Region 4	Region 3	Region 1	Region 2	Wij
Region 5	1	3	5	7	9	0.4988
Region 4	0.33	1	3	5	7	0.2589
Region 3	0.33	0.2	1	3	5	0.1412
Region 1	0.14	0.2	0.33	1	3	0.0669
Region 2	0.11	0.14	0.2	0.33	1	0.0341
Sum	1.91	4.54	9.53	16.33	25	1

Inconsistency rate: 0/066 (due to being less than 0/1 compatibility matrix indices are acceptable)

Table 8. Paired comparison table to the options according to the Educational

Related to Educational	Region 5	Region 4	Region 3	Region 1	Region 2	Wij
Region 5	1	3	5	7	9	0.4988
Region 4	0.33	1	3	5	7	0.2589
Region 3	0.33	0.2	1	3	5	0.1412
Region 1	0.14	0.2	0.33	1	3	0.0669
Region 2	0.11	0.14	0.2	0.33	1	0.0341
Sum	1.91	4.54	9.53	16.33	25	1

Inconsistency rate: 0/066 (due to being less than 0/1 compatibility matrix indices are acceptable)

Table 9. Paired comparison table to the options according to the transportation

Related to Educational	Region 5	Region 4	Region 3	Region 1	Region 2	Wij
Region 5	1	3	5	7	9	0.4988
Region 4	0.33	1	3	5	7	0.2589
Region 3	0.33	0.2	1	3	5	0.1412
Region 1	0.14	0.2	0.33	1	3	0.0669
Region 2	0.11	0.14	0.2	0.33	1	0.0341
Sum	1.91	4.54	9.53	16.33	25	1

Inconsistency rate: 0/066 (due to being less than 0/1 compatibility matrix indices are acceptable)

Table 10. The weight matrix of options according to the criteriatable

Criteria Regions	transportation (Wij)	Educational (Wij)	Area (Wij)	Habitat (Wij)	(Wij) Trade	available (Wij)
Region 5	0.1412	0.1412	0.4988	0.1412	0.1412	0.0669
Region 4	0.4988	0.2589	0.1412	0.4988	0.0341	0.0341
Region 3	0.2589	0.0341	0.2589	0.0341	0.0669	0.1412
Region 1	0.0341	0.4988	0.0341	0.0669	0.2589	0.2589
Region 2	0.0669	0.0669	0.0669	0.2589	0.4988	0.4988

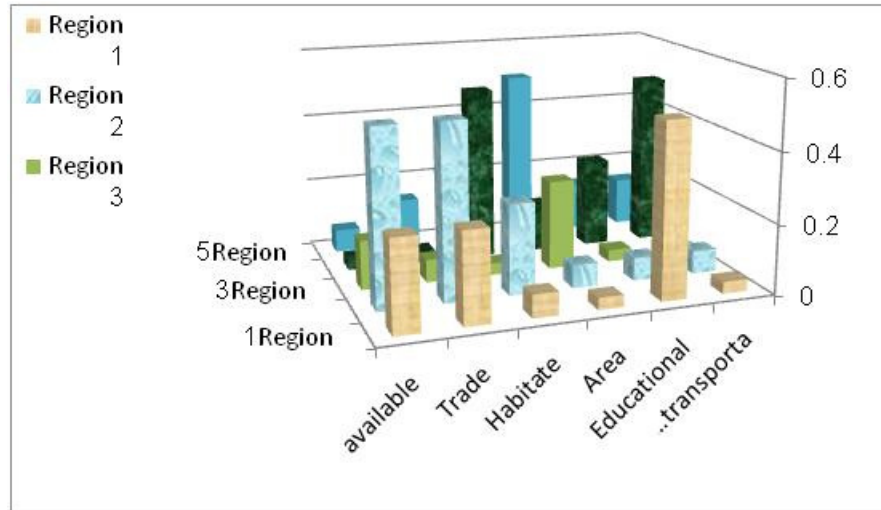


Figure 3. The weight matrix of option according to criteria

Table 11. The weight matrix of criteria according to option

object \ criteria	locating Fire stations (Wij)
available	0.4482
Habitate	0.2542
Trade	0.1461
Area	0.0810
Educational	0.0447
transportation	0.0258

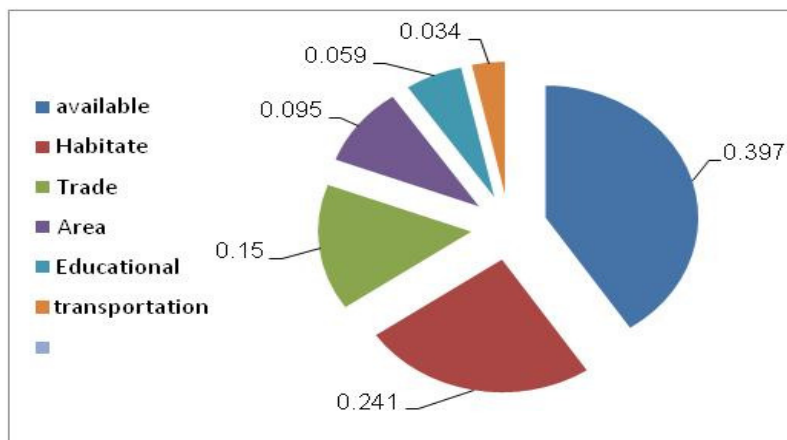


Figure 4. The weight matrix of criteria according to option

Table 12. Points and Ranks

Regions	1	2	3	4	5
point	0/191	0/352	0/171	0/170	0/112
rank	second	first	third	fourth	Fifth

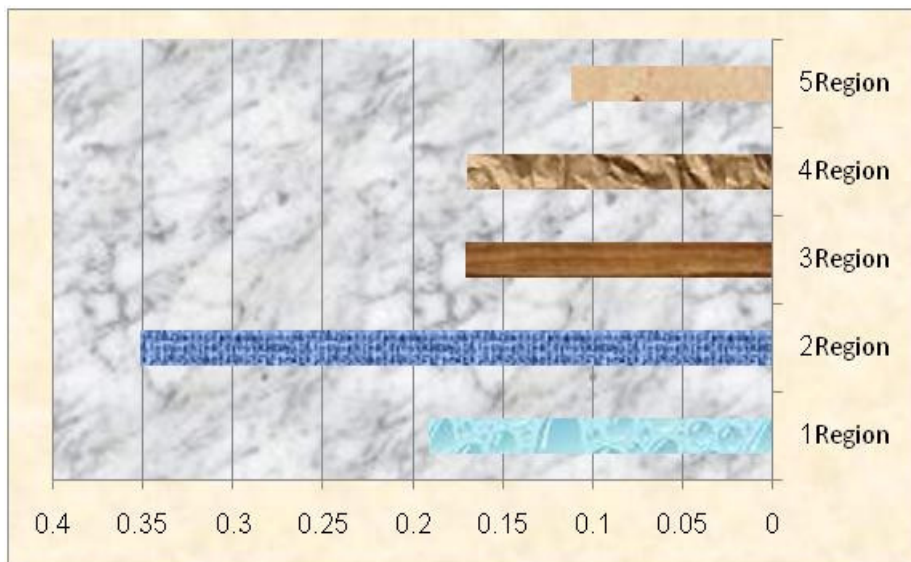


Figure 5. Points and Ranks

Tables (3) to (12) and Figures (3) to (5).

CONCLUSION

It could be stated that using GIS and Analytic Hierarchy Process for analyzing position data and choosing optimized location, has unique performance, so it is suggested that fire fighting organizations use these software applications to create database. It is suggested that fire fighting organization and safety services, create reasonable relationship with urban service centers to enhance their performance, and equip these centers to safety tools before the incident. It is suggested that fire fighting organization and safety services, promote in teaching citizens about how to use fire extinction tools. The three existing stations in Maku city are suitable regarding the population of the city and that one station per 30000 persons is necessary, but because of the vastness of city, these three stations can't service optimally to whole city. It is suggested that 2 additional medium stations to be located in city area and because Maku city has one central station and three secondary (small) stations, and this city doesn't have medium station.

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